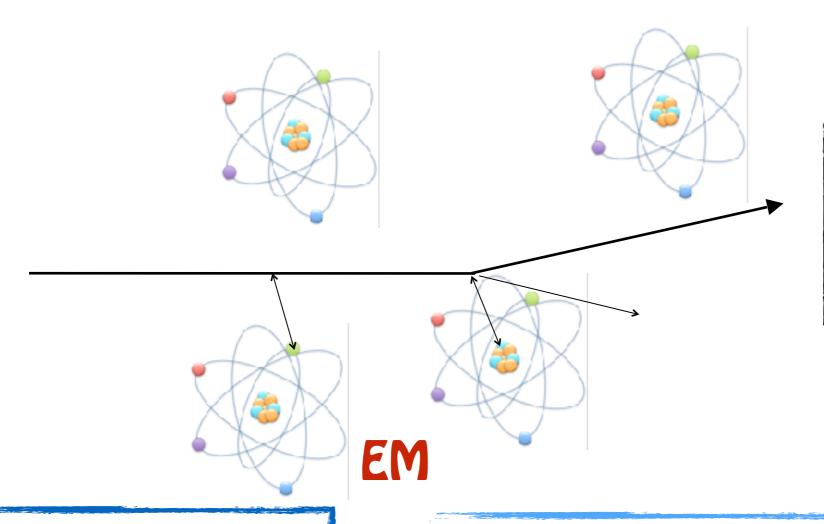
Introduction to interaction of radiation with matter

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Charged Particles

Charged Particles



Charged particles way of interacting is divided in:

- Electrons
- Heavy Charged Particles (m > m_{electron})

Interaction with atomic electrons. Particle loses energy; atoms are **excited** or **ionized**.

Interaction with atomic nucleus. Particle undergoes **multiple scattering**. Could emit a **bremsstrahlung** photon.

Others:

- Cherenkov
- Strong interaction (nuclear fragmentation, ...)



Cross-section

Scattering

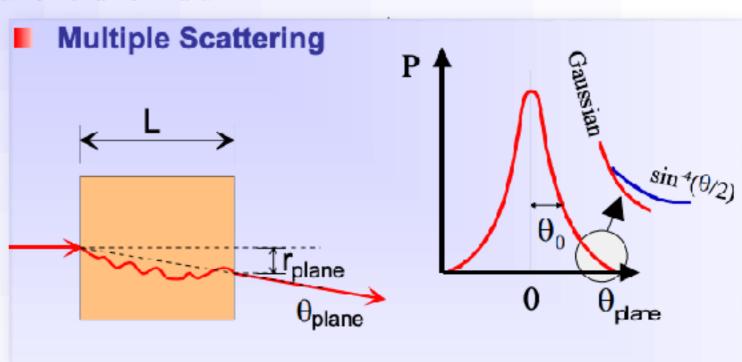
An incoming particle with charge z interacts elastically with a target of nuclear charge Z.

The cross-section for this e.m. process is

$$\frac{d\sigma}{d\Omega} = 4zZr_e^2 \left(\frac{m_e c}{\beta p}\right)^2 \frac{1}{\sin^4 \theta/2}$$

Rutherford formula

- Approximation
 - Non-relativistic
 - No spins
- Average scattering angle $\langle \theta \rangle = 0$
- Cross-section for $\theta \to 0$ infinite!



Scattering does not lead to significant energy loss

Heavy Charged Particles

m >> me

Includes: muons, tau, mesons, hadrons ...

Stopping Power

* Energy loss through ionization and atomic excitation

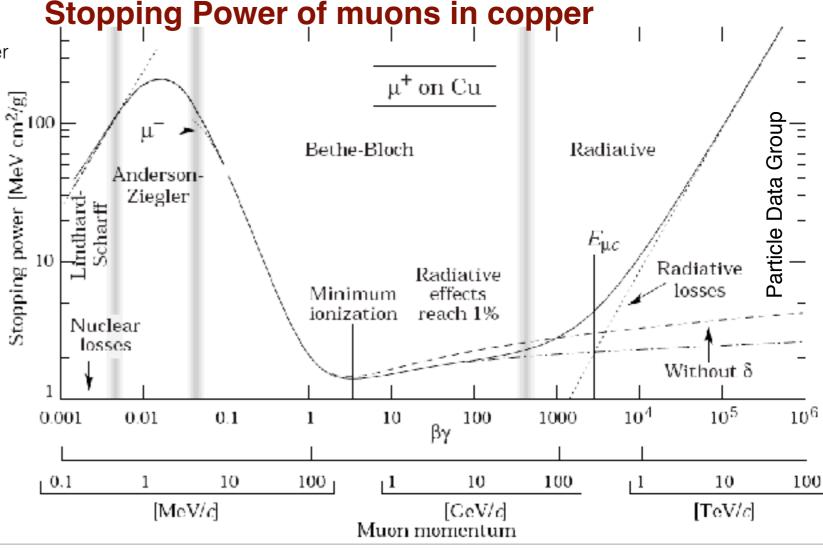
$$\left[-\frac{dE}{dx} = 2\pi N_A r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln \frac{2m_e c^2 \gamma^2 \beta^2 W_{\text{max}}}{I^2} - 2\beta^2 - \delta - 2\frac{C}{Z} \right]$$

Bethe-Bloch Formula

Energy Loss = Energy lost in matter x density

Linear stopping power (S) is the differential energy loss of the particle in the material divided by the differential path length. Also called the specific energy loss.

$$S = -\frac{dE}{dx}$$



Stopping Power

$$-\frac{dE}{dx} = 2\pi N_A r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln \frac{2m_e c^2 \gamma^2 \beta^2 W_{\text{max}}}{I^2} - 2\beta^2 - \delta - 2\frac{C}{Z} \right]$$
Valida per \beta > 0.1

I = potenziale medio di eccitazione

Z, A, ρ = caratteristiche del mezzo

z, β , γ = caratteristiche della particella incidente

 W_{max} = massima energia trasferita in una collisione

 $r_e = raggio classico dell'elettrone <math>r_e = \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{m_e c^2}$..

$$I_{\Theta} = \frac{1}{4\pi\epsilon_0} \cdot \frac{\theta^2}{m_{\theta}c^2} \dots$$

M_e = massa dell'elettrone

 $N_a = N di Avogadro$

C = correzione di shell

 δ = effetto densità

Stopping Power

La capacità di un dato materiale di frenare una particella carica (stopping power) e' definito come la perdita di energia differenziale della particella nel materiale divisa per la lunghezza percorsa in corrispondenza ad essa:

$$S = -\frac{dE}{dx} = \frac{4\pi \, e^4}{m_e} \underbrace{\begin{pmatrix} z^2 \\ v^2 \end{pmatrix}} \rho Z \underbrace{\left[\ln \left(\frac{2m_e v^2}{I} \right) - \ln \left(1 \frac{v^2}{c^2} \right) - \frac{v^2}{c^2} \right]}_{\text{Proprietà del materiale}}$$
(Bethe formula)

v e ze sono la velocità e la carica delle particelle carica pesante incidente

N e Z sono la densità e il numero atomico degli atomi dell'assorbitore

I indica un parametro sperimentale legato all'eccitazione media e al potenziale di ionizzazione

Nota:

- In primissima approssimazione lo stopping power è proporzionale all'inverso dell'energia della
 particella carica incidente (S

 1/E), direttamente proporzionale alla sua carica al quadrato
 (S

 2²) e direttamente proporzionale al numero atomico del materiale (S

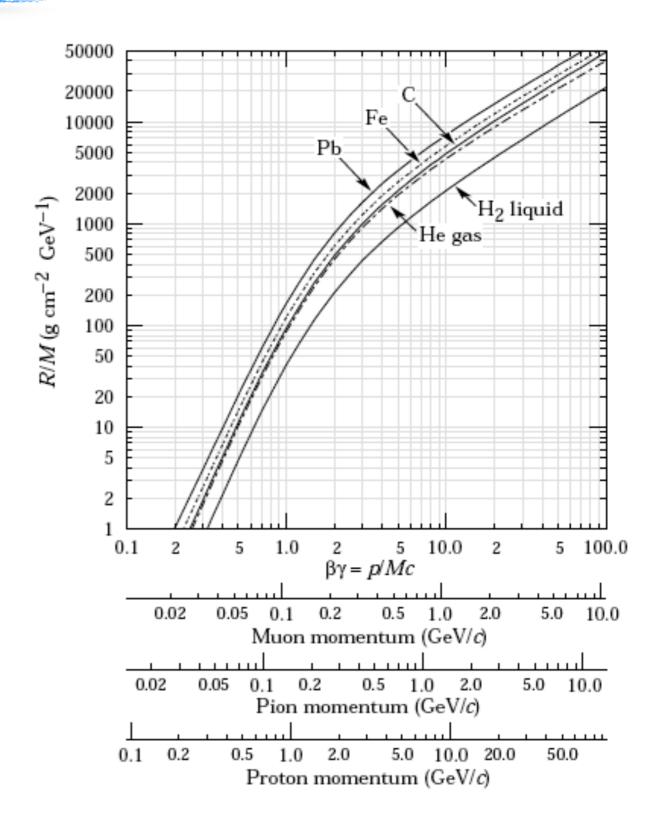
 2)
- Quando più la velocità della particella carica pesante è piccola tanto più è lungo il tempo trascorso nelle vicinanze di un atomo e tanto maggiore sarà l'impulso e l'energia trasferita
- Nel termine in parentesi quadra, solo il primo termine e' significativo in regime non relativistico (v<<c)
- · A bassa energia la particella carica pesante cattura elettroni dagli atomi e quindi si riduce il termine Ze

Range

- * The range is an average quantity.
- * The thickness of material that stops a particle.
- * Integrate the (stopping power)⁻¹ from the Bethe—Bloch formula to obtain the range.

$$R = \int_{E}^{0} \frac{dx}{dE} dE$$

- * Useful for low energy hadrons and muons with momenta below a few hundred GeV (MIP)
- * Radiative Effects important at higher momenta. Additional effects at lower momenta.



Range

Teilchen	in Luft	in Wasser	in Wasser	in Wasser	in Wasser	in Blei
Impuls	I MeV	I MeV	10 MeV	I GeV	I TeV	l TeV
е	3,8 m	4 mm	5 cm			
Р	2,3 cm	30 um			5 km	((als mip: 833 m))
alpha	5 mm	6 um	0,1 mm			
muon				50 m	50 km	833 m

Formule semiempiriche:

Range of alpha in air:

 $R_{alpha_air}[mm] = (0.05E_{alpha} + 2.85)*E^{(3/2)}$ for $E_{alpha} > 4MeV$

Range of protons in air:

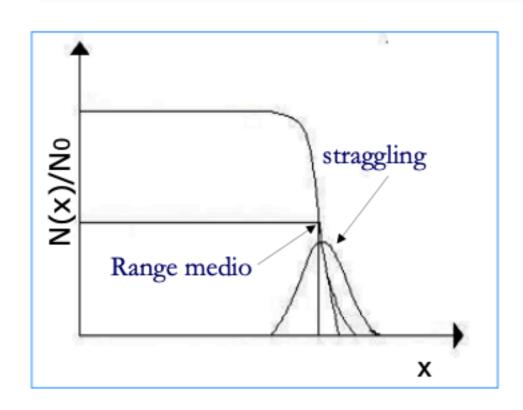
 $R_{proton_air}[m] = (E_{proton}/9.3)^{(1.8)}$ for $E_{proton} < 200 MeV$



Straggling

Il processo di perdita di energia di una particella carica in un materiale è di natura statistica. Se un numero di particelle con la stessa energia attraversa un mezzo, avranno una distribuzione statistica di ranges intorno ad un valor medio, che in prima approssimazione ha forma gaussiana.

Bethe-Bloch: <dE/dx> = valor medio della perdita di energia in uno strato di materia per ionizzazione



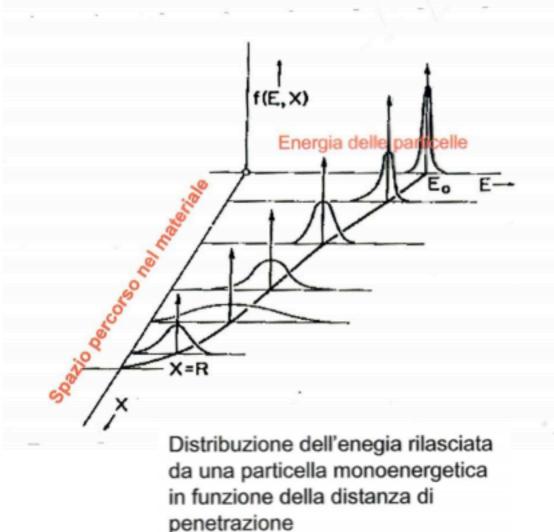
Fluttuazioni statistiche su:

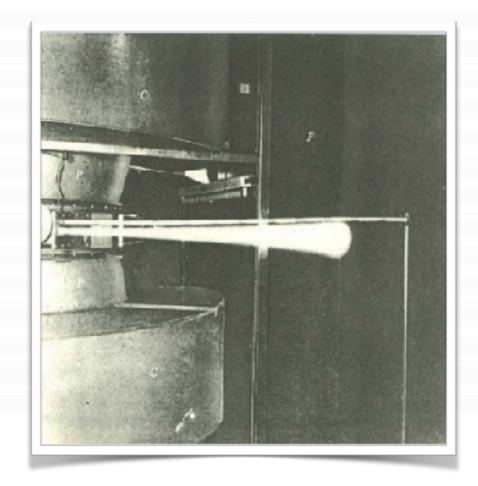
- Numero di collisioni subite
- Energia trasferita per ogni collisione

Straggling

In un fascio di particelle cariche pesanti monocromatiche all'interno di un materiale, l'energia delle particelle e' caratterizzata quindi da una distribuzione di probabilità con un valor medio ed una deviazione standard e/o larghezza a metà altezza.

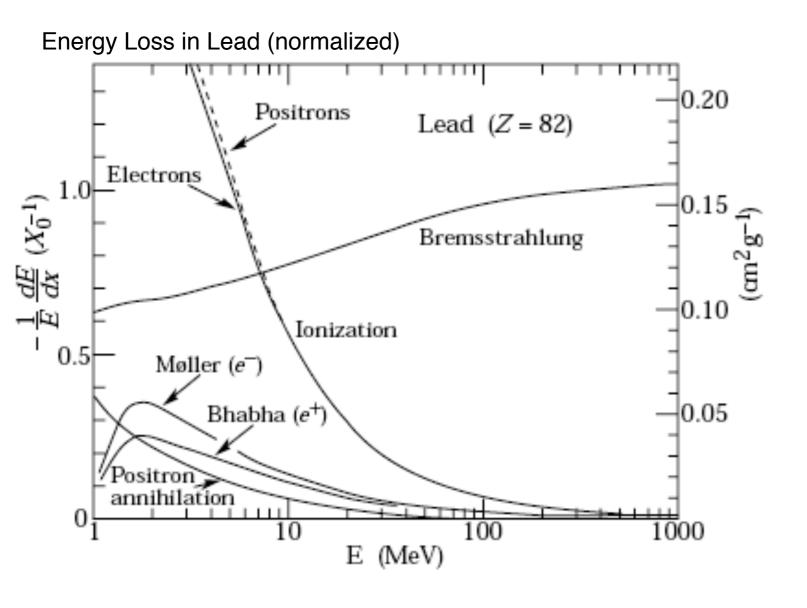
La larghezza a metà altezza della distribuzione di energia varia con la distanza percorsa dalla particella all'interno del materiale assorbente ed è una misura dello straggling





Electrons

Electron Energy Loss



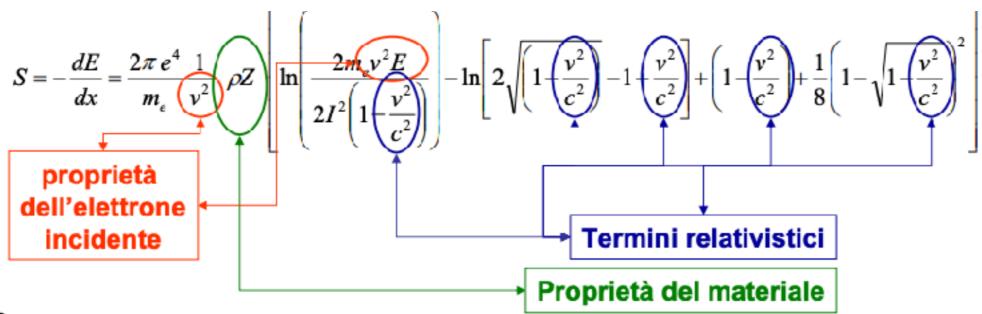
* E' sempre derivato dalla relazione di Bethe, e sostanzialmente ha lo stesso andamento. Assumono più importanza i termini relativistici, associati alla contrazione delle lunghezze

Radiation Length

Mean distance over which an electron loses all but 1/e of its energy.

Electron Energy Loss

Termine Collisionale per l'elettrone (inelastic impact ionization (+Moeller and Bhabba scattering))



Dove:

v è la velocità dell'elettrone incidente
 N e Z sono la densità e il numero atomico degli atomi dell'assorbitore
 / indica un parametro sperimentale legato all'eccitazione media e al potenziale di ionizzazione

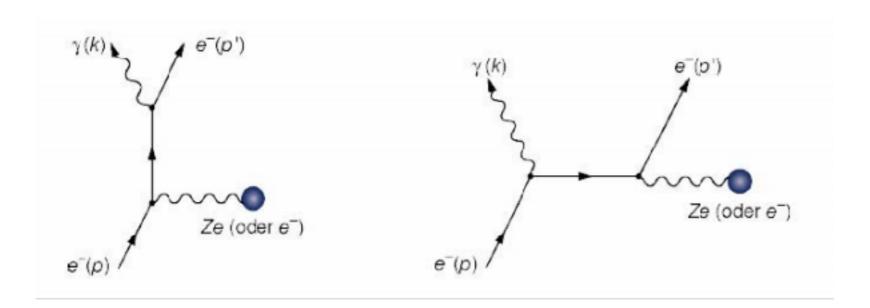
Nota:

- In primissima approssimazione lo stopping power è direttamente proporzionale al numero atomico del materiale (S

 Z)
- Quanto più la velocità dell'elettrone è piccola tanto più è lungo il tempo trascorso nelle vicinanze di un atomo e tanto maggiore sarà l'impulso e l'energia trasferita
- · Nel termine in parentesi quadra solo il primo termine e' significativo in regime non relativistico (v<<c)

Bremsstrahlung

- * A charged particle of mass M and charge q is deflected by a nucleus of charge Ze (charge partially shielded by electrons)
- * The deflection accelerates the charge and therefore it radiates -> bremsstrahlung



Presence of nucleus required for the conservation of energy and momentum

Beachte:

$$-\left(\frac{dE}{dx}\right)_{\text{rad}} \propto E \quad \text{und} \quad -\left(\frac{dE}{dx}\right)_{\text{rad}} \propto \frac{1}{m^2}$$

Attenzione, gia per il muone (seconda particella piu leggera) la perdita di energia d'ovuto alla Bremsstrahlung e 40000 volte piu piccola rispetto al elettrone

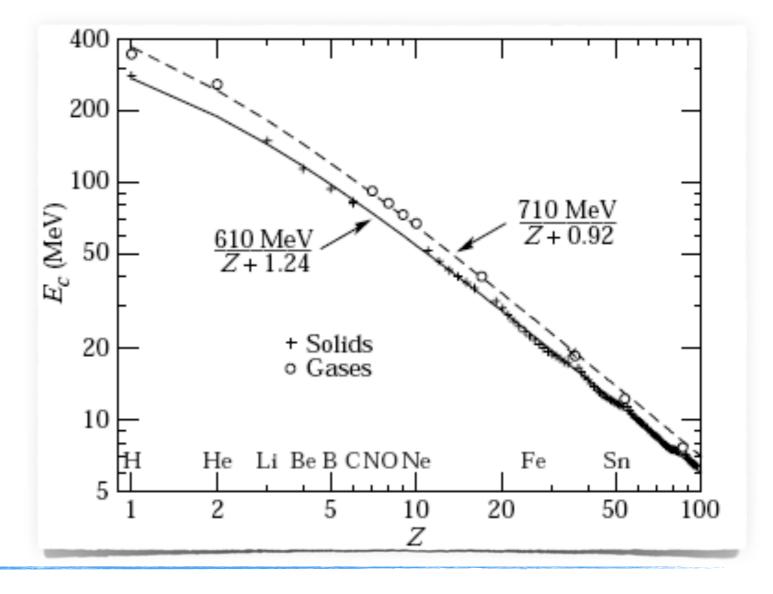
Electron Critical Energy

- * Energy loss through bremsstrahlung is proportional to the electron energy
- * Ionization loss is proportional to the logarithm of the electron energy
- * Critical energy (Ec) is the energy at which the two loss rates are equal

$$E_c = \frac{800 \text{MeV}}{Z + 1.2}$$

Electron in Copper: $E_c = 20 \text{ MeV}$

Muon in Copper: $E_c = 400 \text{ GeV}!$

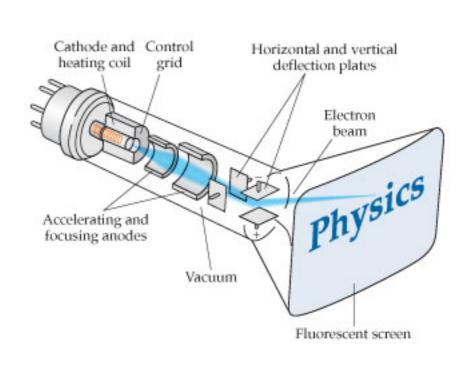


Detection: Tracking Detectors

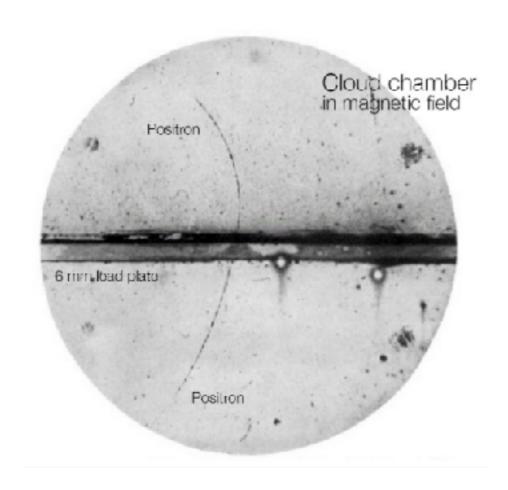
Tracking Detectors

- * Based on detecting ionisation caused by passage of the particle
- * Different ways of doing it
- * In common: Tracking detectors has fine granularity, low energy deposition and are often in a magnetic field



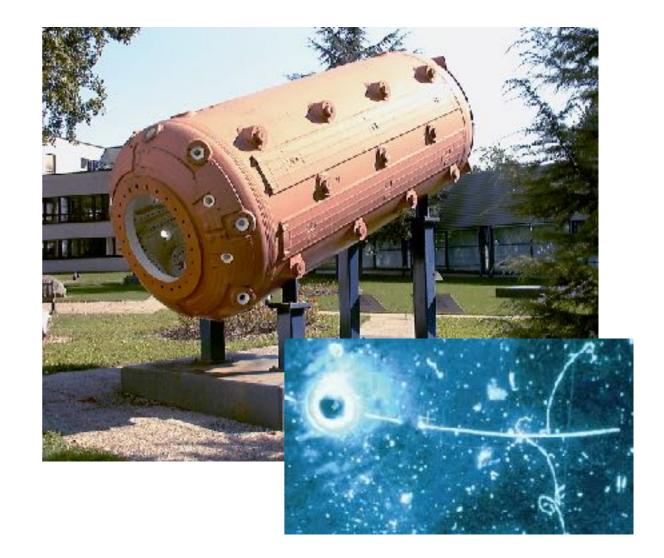


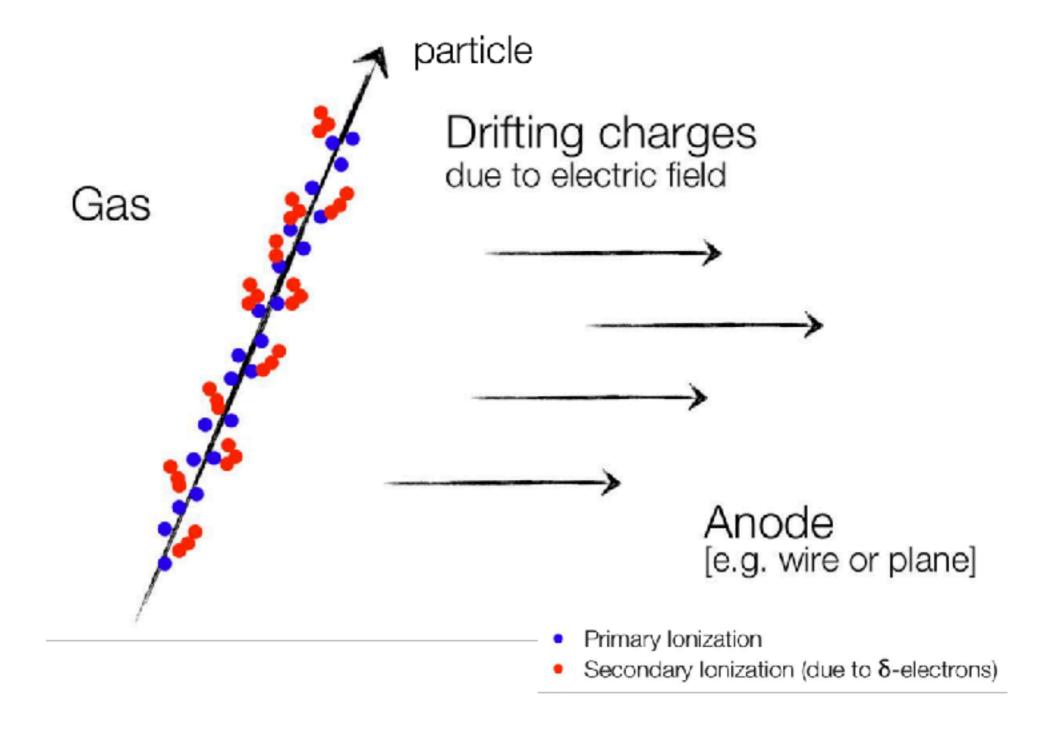
Bubble and Cloud Chamber



A supersaturated vapor of water or alcohol that form tiny drops around ionised molecules (condensation centers)

liquid heated to just below its boiling point, decreasing pressure it become metastable. Bubbles forms around ionisation points.

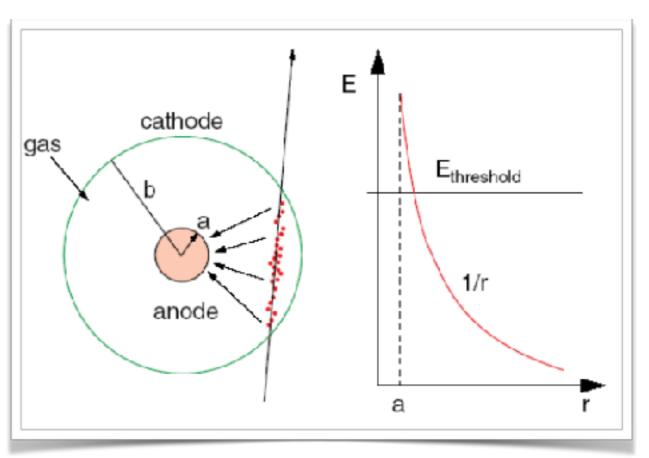




- * Electrons liberated by ionization drift towards the anode wire.
- * Electrical field close to the wire (typical wire Ø ~few tens of µm) is sufficiently high for electrons (above 10 kV/cm) to gain enough energy to ionize further

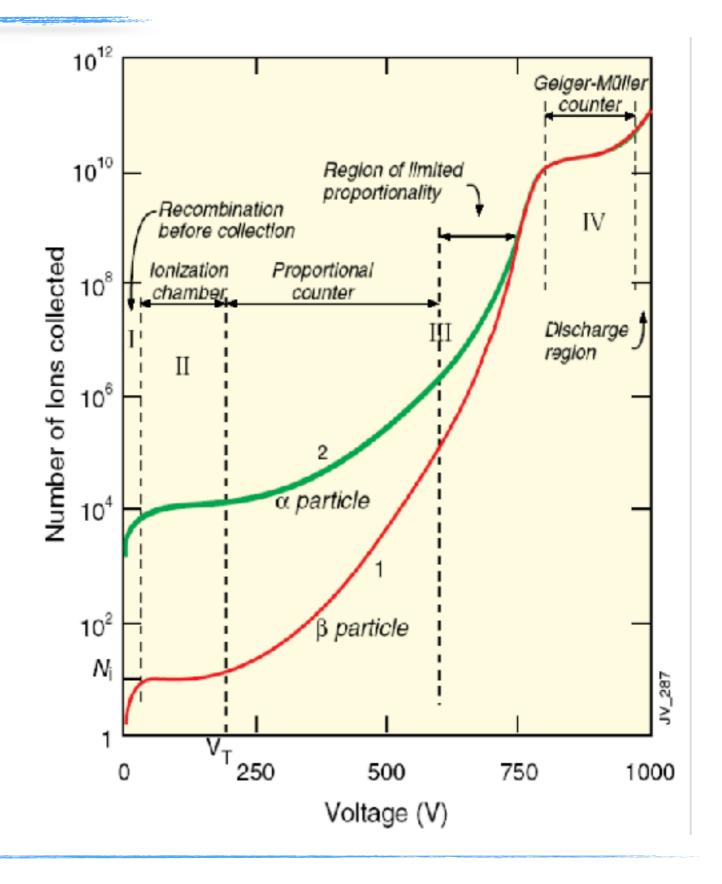
$$E(r) = \frac{CV_0}{2\pi\epsilon_0} \cdot \frac{1}{r} \qquad V(r) = \frac{CV_0}{2\pi\epsilon_0} \cdot \ln\frac{r}{a}$$

* Avalanche — exponential increase of number of electron ion pairs.



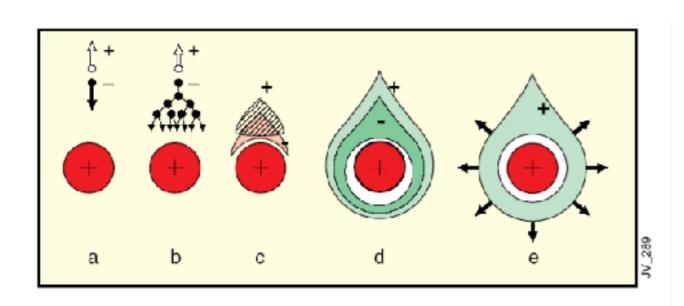


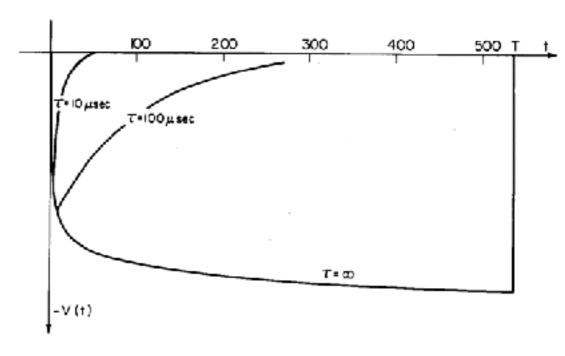
- ionization mode full charge collection, but no charge multiplication; gain ~ 1
- * proportional mode multiplication of ionization starts;
 - signal proportional to original ionization
 → possible energy measurement (dE/dx);
 - secondary avalanches have to be quenched; gain ~ 10⁴ – 10⁵
- * limited proportional mode (saturated, streamer) strong photoemission;
 - secondary avalanches merging with original avalanche;
 - requires strong quenchers or pulsed HV; Large signals → simple electronics; gain ~ 10¹⁰
- Geiger mode massive photoemission;
 - full length of the anode wire affected; discharge stopped by HV cut; strong quenchers needed as well



Signal Formation

- * Avalanche formation within a few wire radii and within t < 1 ns!
- * Signal induction both on anode and cathode due to moving charges (both electrons and ions).
- * Electrons collected by anode wire, i.e. dr is small (few µm). Electrons contribute only very little to detected signal (few %).
- * Ions have to drift back to cathode







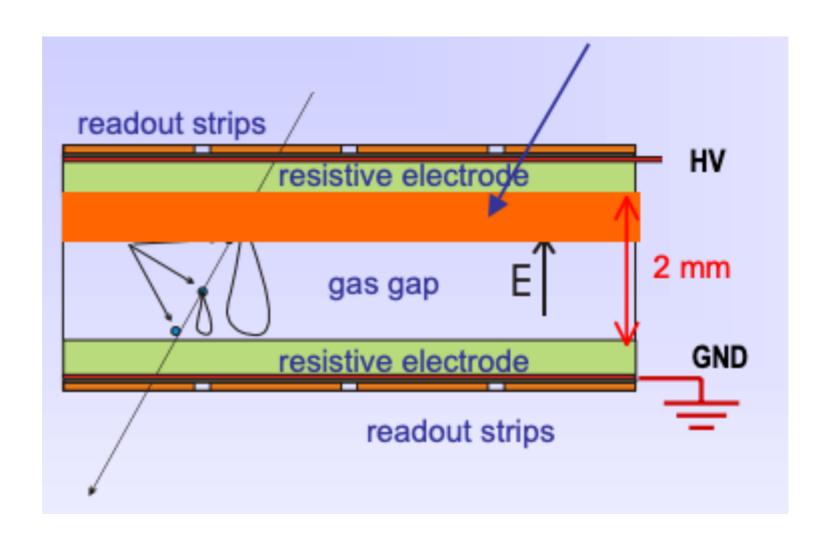
Multiwire Proportional Chamber

Typical geometry

- between plates5mm
- 1mm between wires,
- radius 20 µm



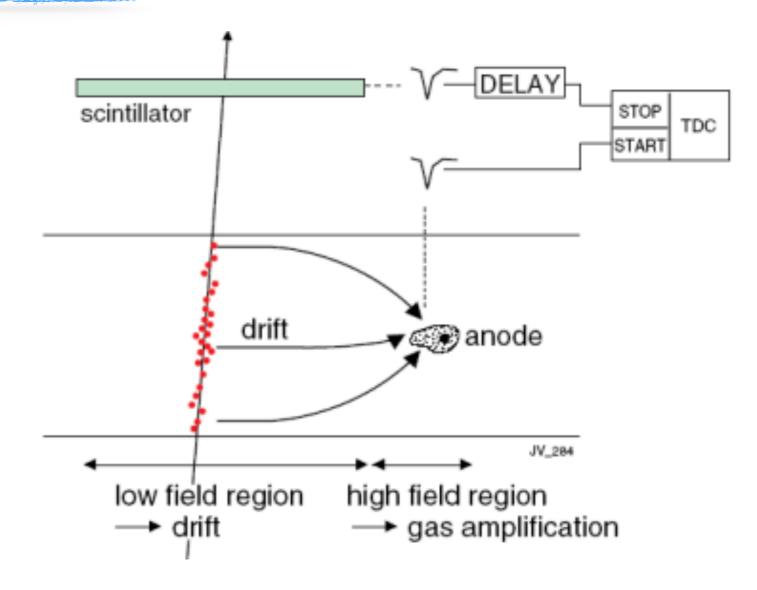
RPC - Resistive Plate Chamber



exceptional time resolution suited for the trigger applications

Drift Chambers

- * Measure arrival time of electrons at sense wire relative to a time to.
- * Need a trigger (bunch crossing or scintillator).
- * Drift velocity independent from E.



Resolution determined by diffusion, primary ionization statistics, path fluctuations and electronics

Gas Detectors in LHC Experiments

ALICE: TPC (tracker), TRD (transition rad.), TOF (MRPC), HMPID (RICH-pad chamber),

Muon tracking (pad chamber), Muon trigger (RPC)

ATLAS: TRD (straw tubes), MDT (muon drift tubes), Muon trigger (RPC, thin gap chambers)

CMS: Muon detector (drift tubes, CSC), RPC (muon trigger)

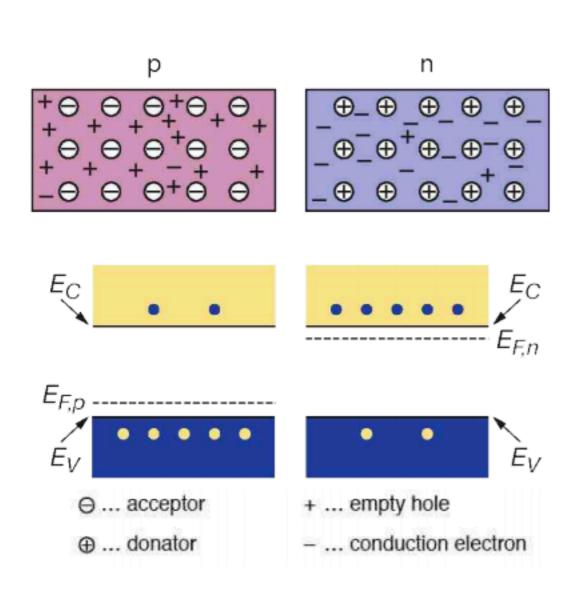
LHCb: Tracker (straw tubes), Muon detector (MWPC, GEM)

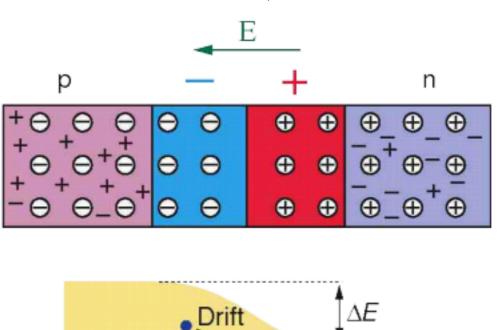
TOTEM: Tracker & trigger (CSC, GEM)

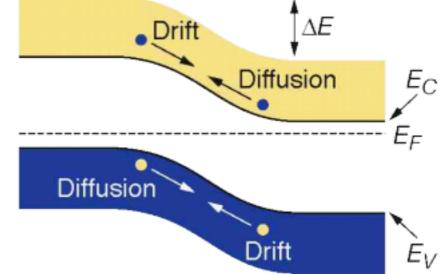


Silicon Detectors

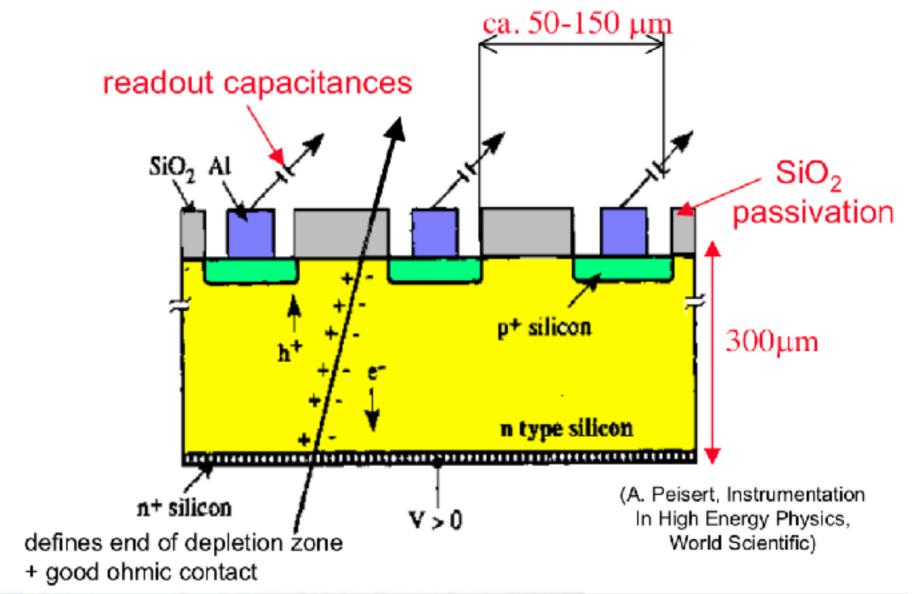
* Between n—type and p—type semiconductor a stable region free of charge carries is created and is called the depletion zone







Silicon Detectors



High granularity, very sensible, low voltage, high price for large coverage Used in high track density regions

Last Lesson Summary

Ionisation VS Bremsstrahlung

Bethe-Bloch Formula

$$-\frac{dE}{dx} = 2\pi N_A r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln \frac{2m_e c^2 \gamma^2 \beta^2 W_{\text{max}}}{I^2} - 2\beta^2 - \delta - 2\frac{C}{Z} \right]$$

$$- \left\langle \frac{dE}{dx} \right\rangle_{\rm el.} = \ K \frac{Z}{A} \frac{1}{\beta^2} \left[\ln \frac{m_e \beta^2 c^2 \gamma^2 T}{2I^2} \ + \ F(\gamma) \right] \label{eq:el.}$$

[T: kinetic energy of electron]

$$W_{max} = \frac{1}{2}T$$

*
$$\propto ln(E)$$

* Basse energie/impulsi

$$-\left\langle rac{dE}{dx}
ight
angle pprox rac{4N_aZ^2lpha^3(\hbar c)^2}{m_e^2c^4}E\lnrac{183}{Z^{1/3}}$$

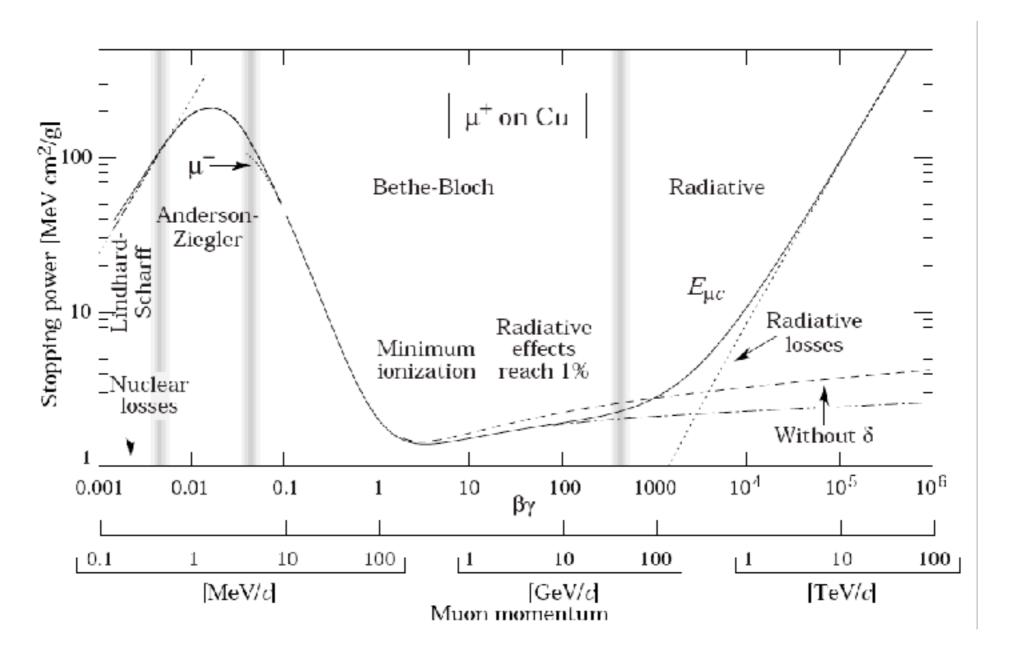
*
$$\propto Z^2$$

*
$$\propto E/m^2$$

* Alte energie/impulsi

Heavy Charged

$$\left(\frac{dE}{dx}\right)_{\mathrm{Tot}} = \left(\frac{dE}{dx}\right)_{\mathrm{Ion}} + \left(\frac{dE}{dx}\right)_{\mathrm{Brems}}$$



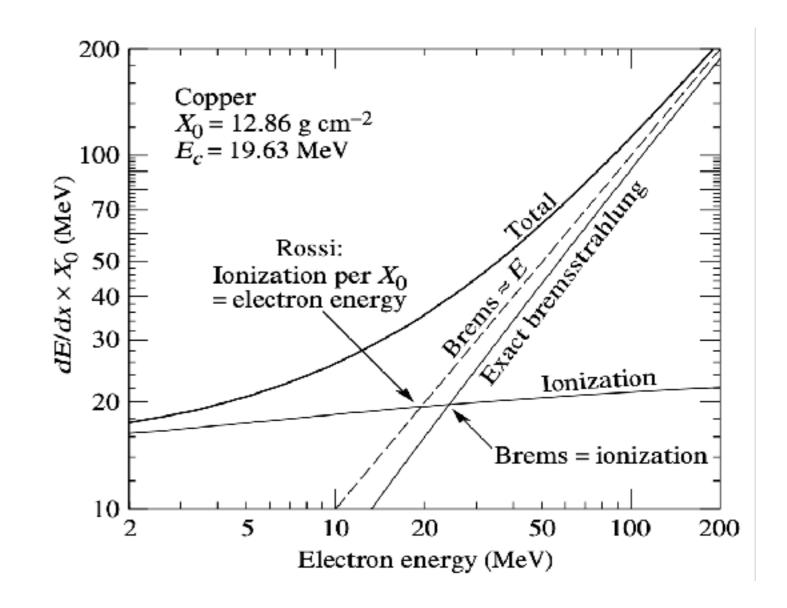


Electrons

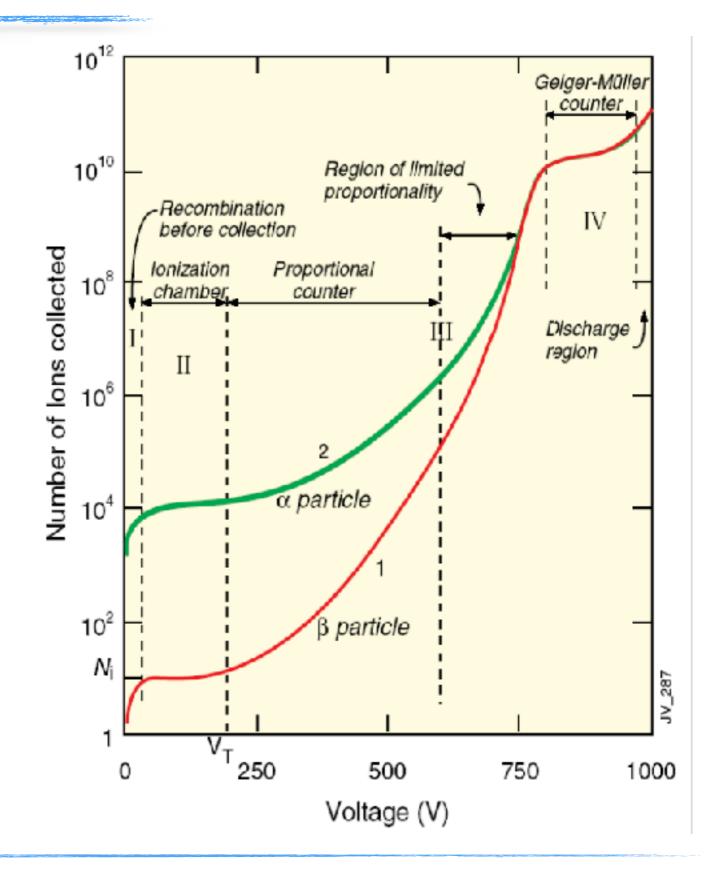
$$\left(\frac{dE}{dx}\right)_{\mathrm{Tot}} = \left(\frac{dE}{dx}\right)_{\mathrm{Ion}} + \left(\frac{dE}{dx}\right)_{\mathrm{Brems}}$$

Critical Energy

$$E_c = \frac{800 \text{MeV}}{Z + 1.2}$$



- ionization mode full charge collection, but no charge multiplication; gain ~ 1
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Tracker Comparison

- * Multiwire chamber:
 - resolution ~0,5 mm
 - time ~10 ns (quite fast)

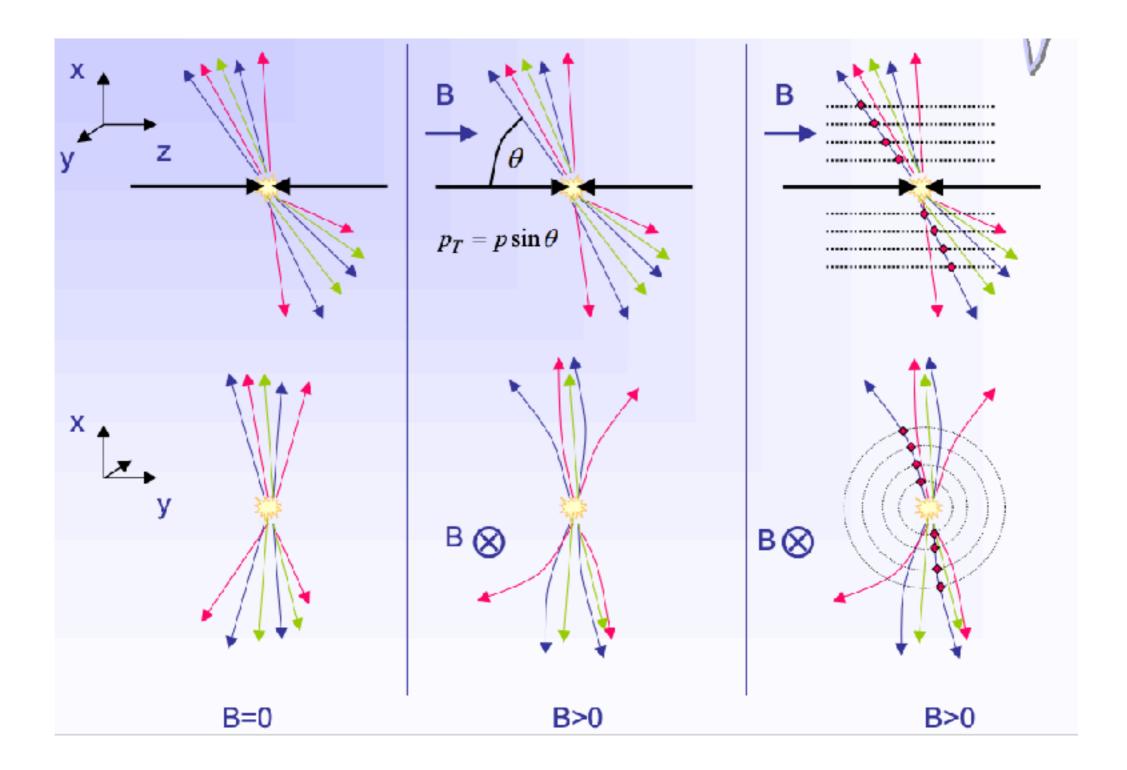
- * RPC:
 - fast!
 - No great spatial resolution
 - Used as trigger

- * Drift Chamber:
 - 9 high resolution ~40 μm
 - expansive!
 - Good at low rate

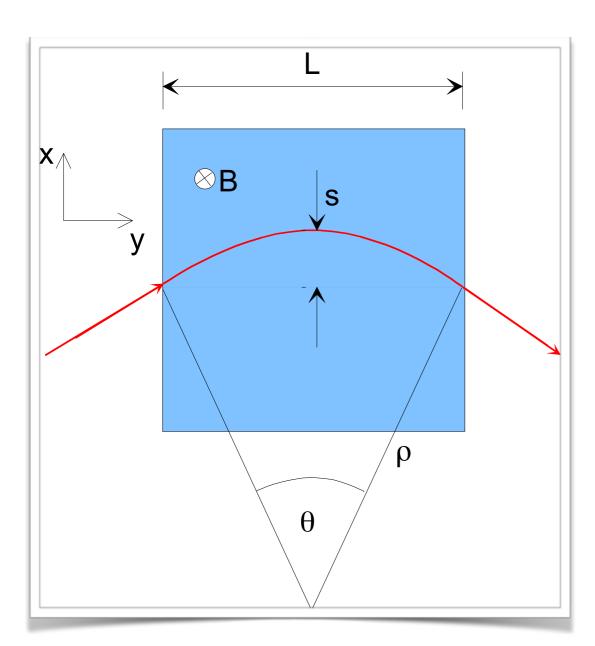


Momentum Measurement

Momentum Measurement



Momentum Measurement



$$\frac{mv^2}{\rho} = q(v \times B) \rightarrow p_T = qB\rho \qquad p_T \text{ (GeV/c)} = 0.3B\rho \text{ (T·m)}$$

$$\frac{L}{2\rho} = \sin\theta/2 \approx \theta/2 \quad \Rightarrow \quad \theta \approx \frac{0.3L \cdot B}{p_T}$$

$$s = \rho (1 - \cos\theta/2) \approx \rho \frac{\theta^2}{8} \approx \frac{0.3}{8} \frac{L^2 B}{p_T}$$

For the simple case of three measurements: $s = x_2 - (x_1 + x_3)/2 \Rightarrow ds = dx_2 - dx_1/2 - dx_3/2$ with $\sigma_x \approx dx_i$ uncorrelated error of single measurement:

$$\sigma_s^2 = \sigma_x^2 + \frac{\sigma_x^2}{4} \cdot 2 = \frac{3}{2} \sigma_x^2$$

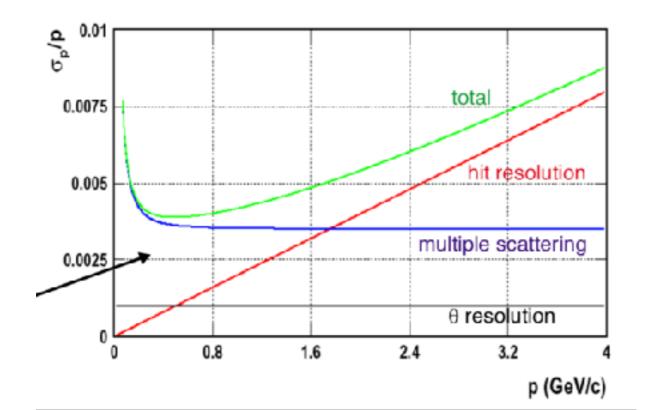
Momentum Resolution

$$\frac{\sigma(p_T)}{p_T} = \frac{\sigma_s}{s} = \sqrt{\frac{3}{2}}\sigma_x \cdot \frac{8p_T}{0.3BL^2}$$

- degrades linearly with transverse momentum
- · improves linearly with increasing B field
- · improves quadratically with radial extension of detector

$$\frac{\sigma(p_T)}{p_T} = \frac{\sigma(\kappa)}{\kappa} = \frac{\sigma_x \cdot p_T}{0.3BL^2} \sqrt{\frac{720}{(N+4)}}$$

In the case of N equidistant measurements



Adding the contribution of the multiple scattering

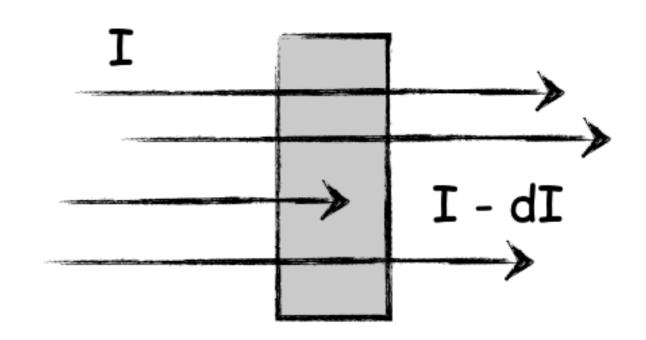
$$\left(\frac{\sigma_p}{p}\right)^2 = \left(\sqrt{\frac{720}{N+4}} \frac{\sigma_x p \sin \theta}{0.3BL^2}\right)^2 + \left(\frac{0.2}{\beta B \sqrt{LX_0 \sin \theta}}\right)^2$$

Photons &

Interaction of photons with matter

Characteristic for interactions of photons with matter:

A single interaction removes photon from beam!

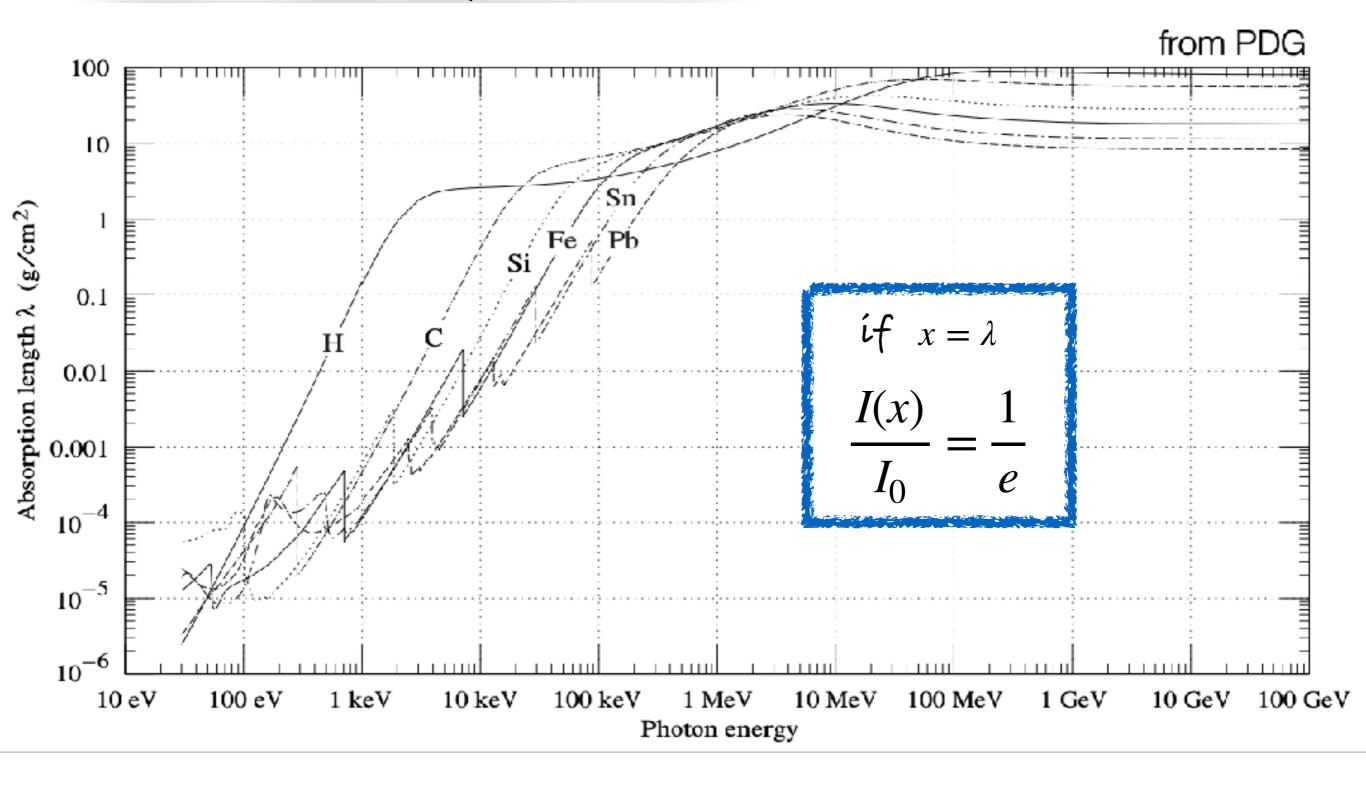


$$dI = -\mu\,I\,dx \\ [\,\mu\colon \text{absorption coefficient}\,] \\ \text{depends on} \\ \text{E, Z, } \rho$$

→ Beer-Lambert law:

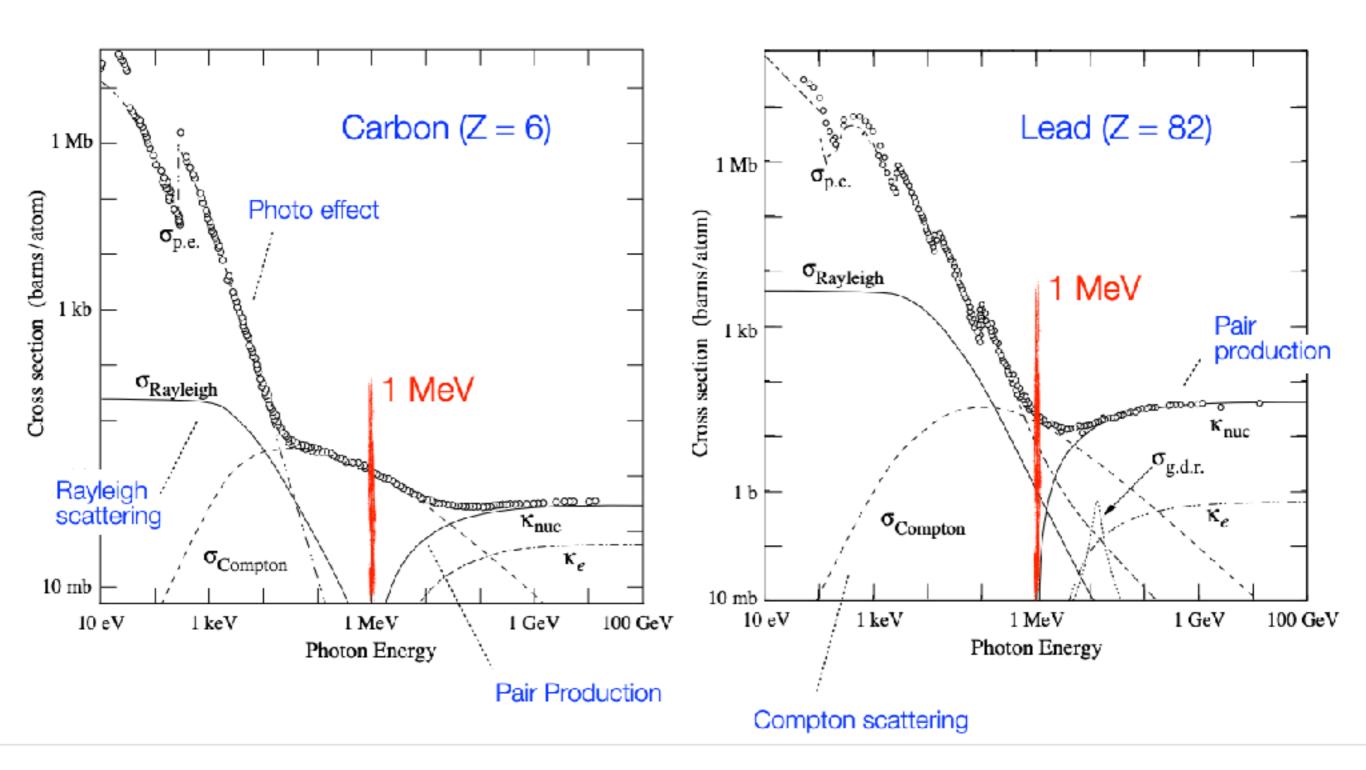
$$I(x) = I_0 e^{-\mu x}$$
 with $\lambda = 1/\mu = 1/n\sigma$ [mean free path]

Mean Free path





Photon Total Cross Sections





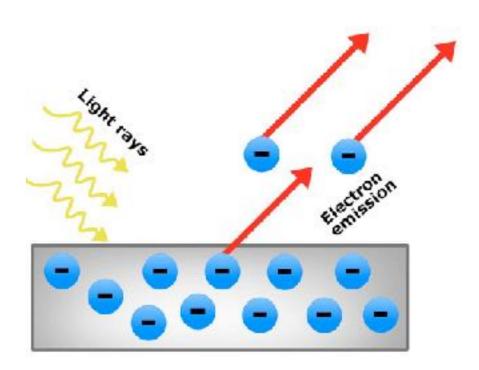
Photoelectric Effect



- * The photon is absorbed by the atom, an electron is emitted (ionization)
- * Photon energy > binding energy (~20eV)
 - If not is Auger effect



$$\sigma_{photo} = 4\alpha^4 \sqrt{2} Z^5 \frac{8\pi r_e^2}{3} \left(\frac{m_e c^2}{h v}\right)^{7/2}$$

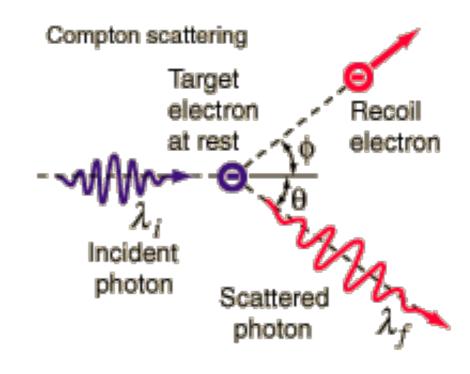


 $\infty E^{7/2}$

 \propto Z⁴ or Z⁵

Compton

- * X—rays interact directly with the atomic electrons
- * Higher energy than photoelectric photons
- * $\lambda_i \approx 0.02 \text{ nm}$

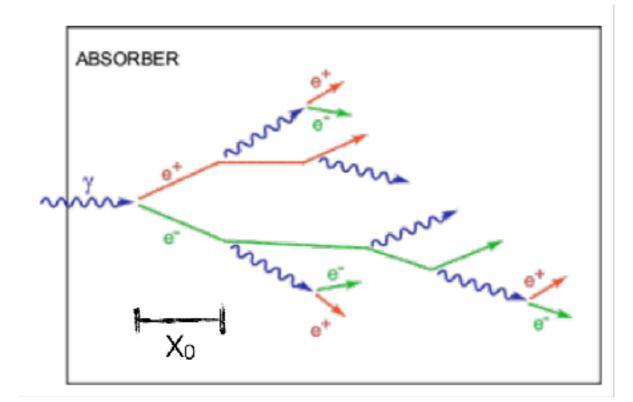


$$\lambda_f - \lambda_i = \Delta \lambda = \frac{h}{m_e c} (1 - \cos \theta)$$

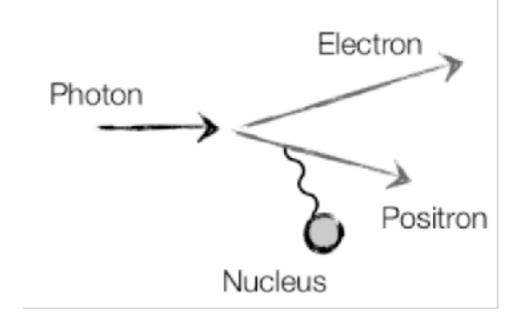
Photon Pair Production

Cross Section: [for E_Y » m_ec²]

$$\sigma_{\rm pair} \approx \frac{7}{9} \left(4 \, \alpha r_e^2 Z^2 \ln \frac{183}{Z^{\frac{1}{3}}} \right)$$



$$E_{th} = 2me = 1,022 \text{ keV}$$



$$\lambda$$
 = mean free path

$$\lambda = \frac{9}{7}X_0$$

Xo = radiation length = mean distance over which an electron loses all but 1/e of its energy.

Longitudinal Shower Profile

- * After each interaction the particle energy halves
- * Shower stops when the energy equals E_{th}

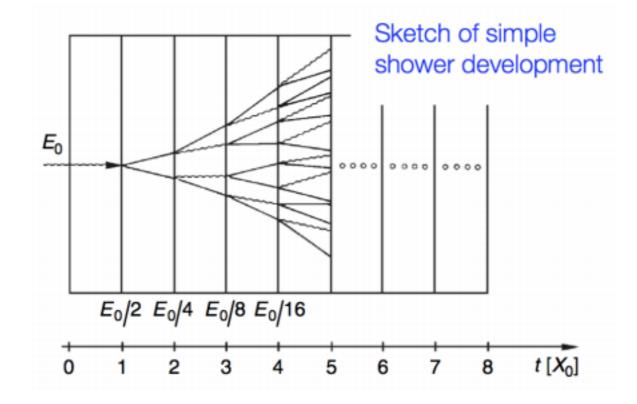
Number of shower particles after depth t:

$$N(t) = 2^t$$

Energy per particle after depth t:

$$E = \frac{E_0}{N(t)} = E_0 \cdot 2^{-t}$$

$$\rightarrow t = \log_2(E_0/E)$$



Total number of shower particles with energy E₁:

$$N(E_0,E_1)=2^{t_1}=2^{\log_2(E_0/E_1)}=rac{E_0}{E_1}$$

Number of shower particles at shower maximum:

$$N(E_0, E_c) = N_{\text{max}} = 2^{t_{\text{max}}} = \frac{E_0}{E_c}$$

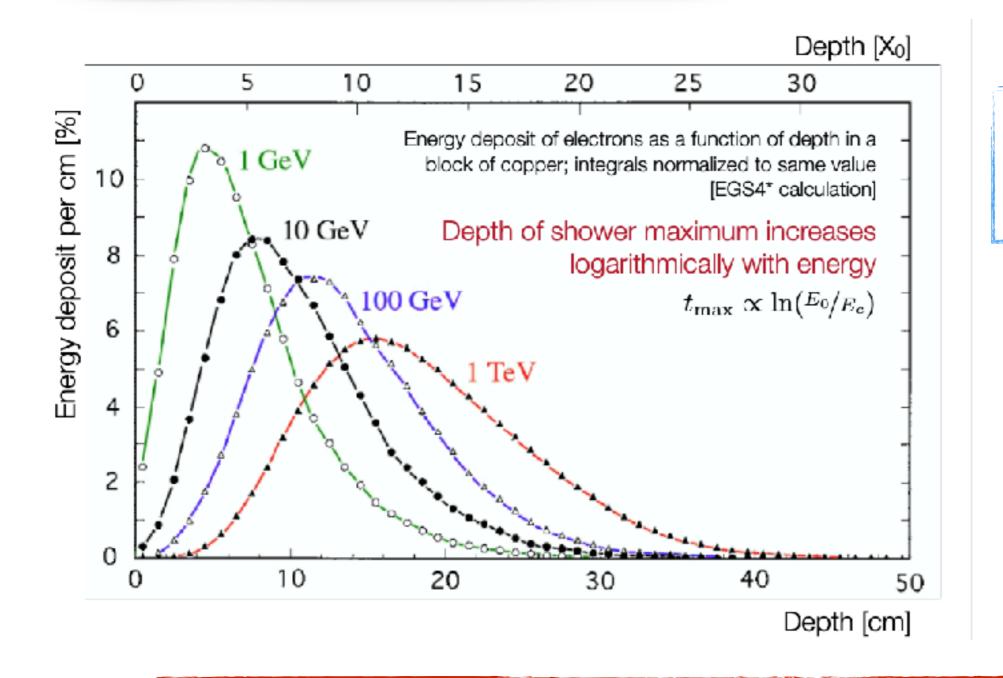
 $\propto E_0$

Shower maximum at:

$$t_{
m max} \propto \ln(E_0/E_c)$$



Longitudinal Shower Profile



Longitudinal development scales with the radiation length

Electrons eventually fall beneath critical energy and then lose further energy through dissipation and ionization

Transverse Shower Profile

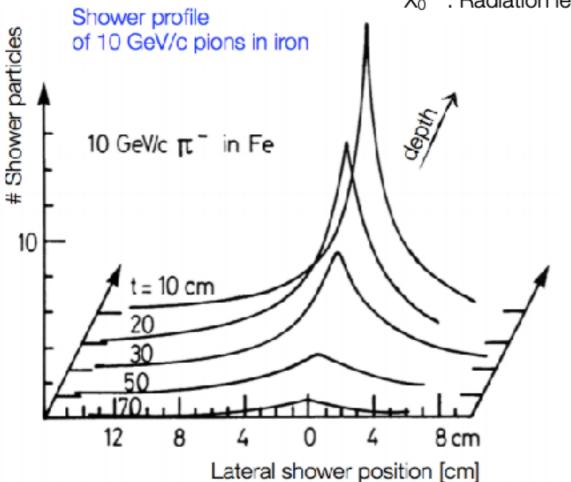
Transverse size of EM shower given by radiation length via Molière radius

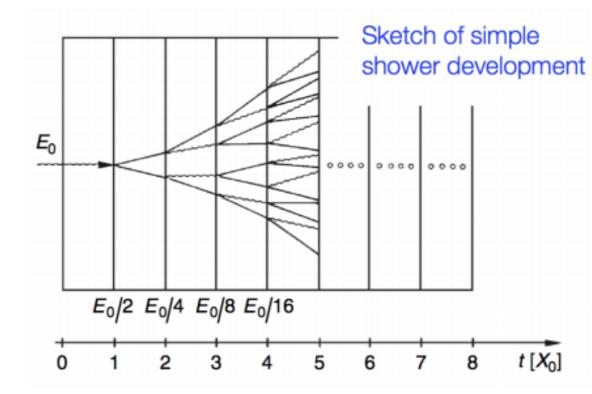
$$R_M = \frac{21 \text{ MeV}}{E_c} X_0$$

R_M: Moliere radius

Ec : Critical Energy [Rossi]

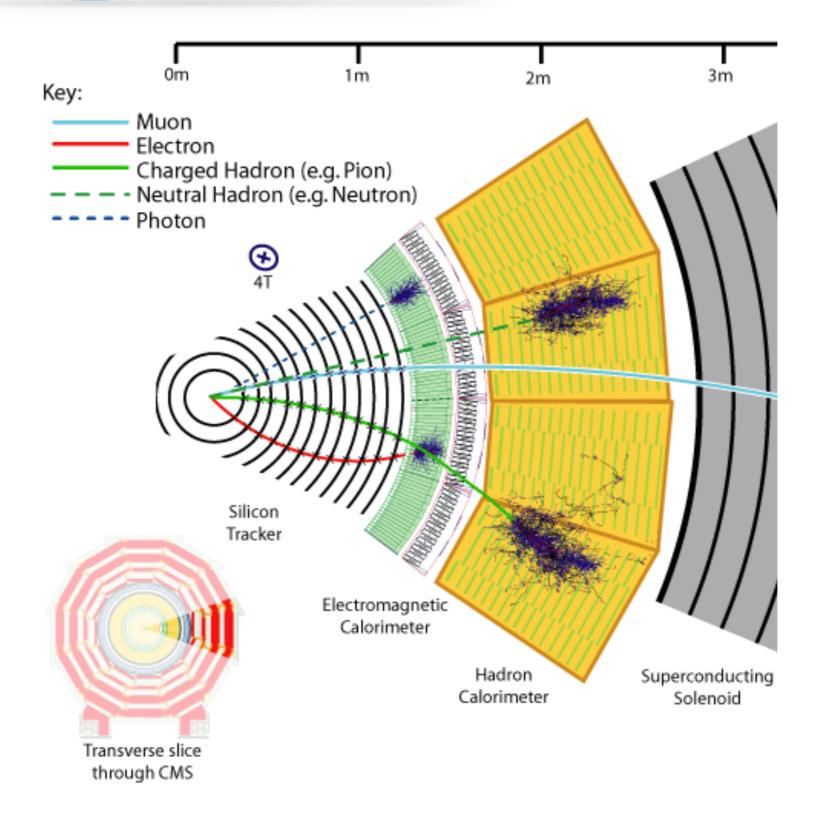
X₀ : Radiation length





Transverse development scales with the radiation length

Electromagnetic Cascades



Photon (and Electron) Detection

N.B.: it interacts with all other charged particles anyway

* In high energy physics we are interested at high energies (E_{γ} >> MeV)

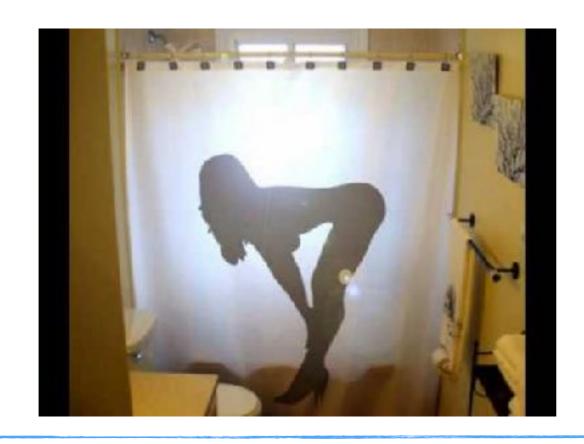


- * Photons interacts mostly by Pair Production
- * We are interested in revealing showers

* In high energy physics we are interested at high energies (E_{γ} >> MeV)

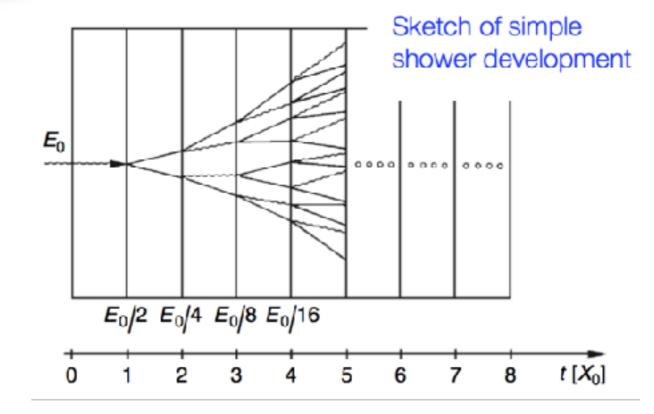


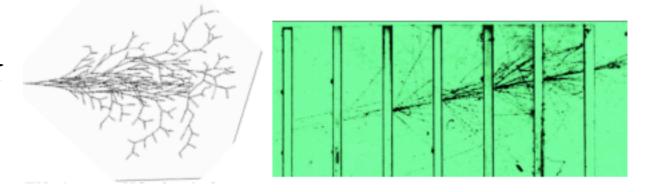
- * Photons interacts mostly by Pair Production
- * We are interested in revealing showers





- * Calorimeters look at particle showers
- * Shower is a complex object formed by multiple particles
- * Study the shower property to understand the characteristic of the initial particle (e or γ)
- * Strategy: look at the energy lost by electrons in the shower by ionisation
- * Electron sensitive detector needed





* Longitudinal depth used to evaluate the particle energy

$$t_{\max}[X_0] \sim \ln \frac{E_0}{E_c}$$

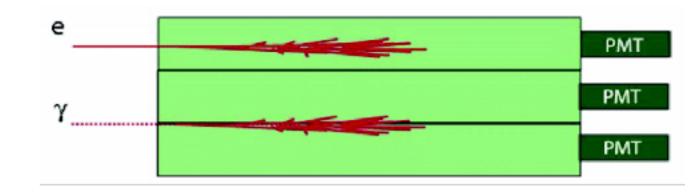
- * So it must be longitudinally segmented!
- * Transverse segmentation gives info on the particle position and direction (for charged particles trackers are better)
- * The energy resolution (uncertainty) depends on the inverse of sqrt(number of steps t) for Poisson. But t proportional to the initial energy, so...

$$\frac{\sigma(E)}{E} = \frac{A}{\sqrt{E}}$$

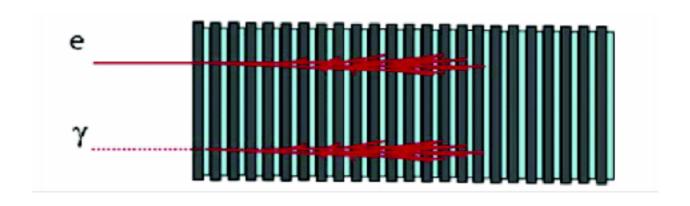
* N.B.: to have the total energy the shower must be fully contained in the calorimeter

- * initiate an em shower (high Z)
- * collect shower energy (light) lost by ionization

- * homogeneous: the sensitive detector is the same that causes the em shower.
 - based on crystal (BGO, NaI, Pb-glass),
 high density,
 - excellent resolution, expensive



- * **Sampling:** alternating layers of passive and sensitive material
 - gas, silicon (sensitive)
 - Passive dense material (Fe, Pb, W, ...),
 - poorer resolution, compact, cheap

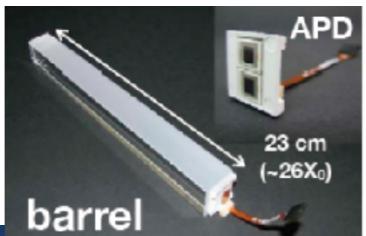




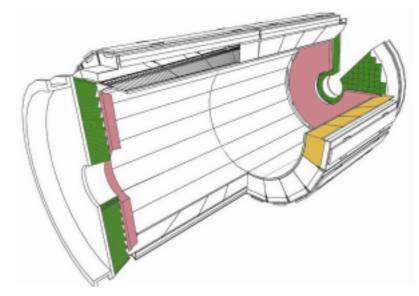
Examples of Homogeneous Calo

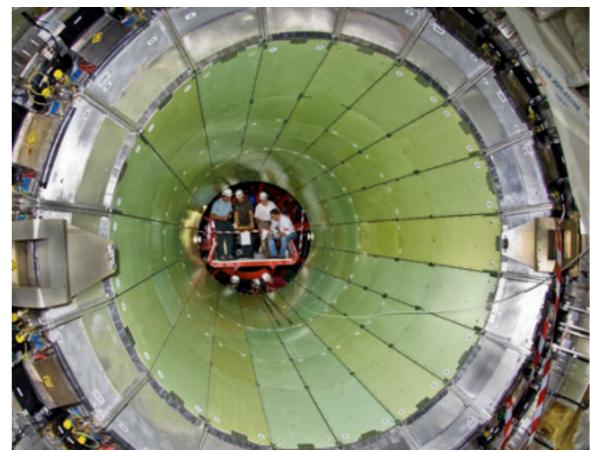
CMS ECal

- Scintillator: PbWO
- ~3x3x23 cm
- 70000 crystals



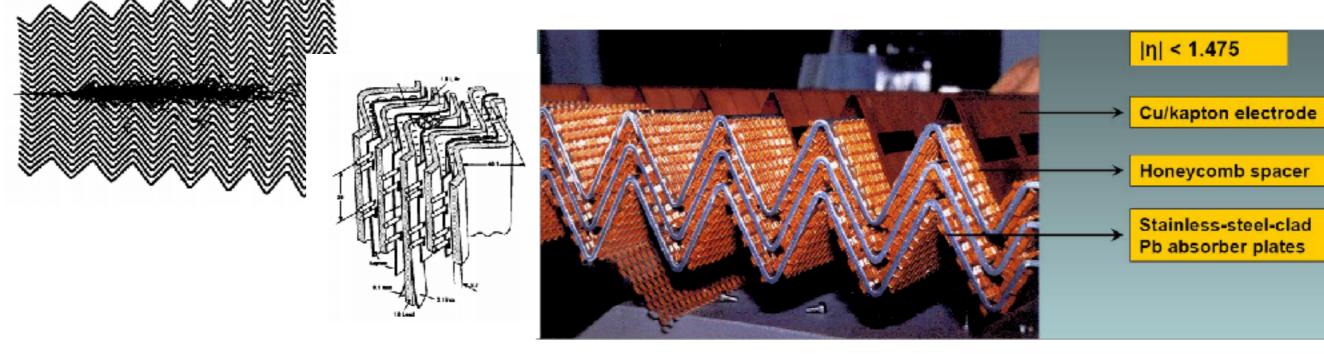






Examples of Sampling Calo



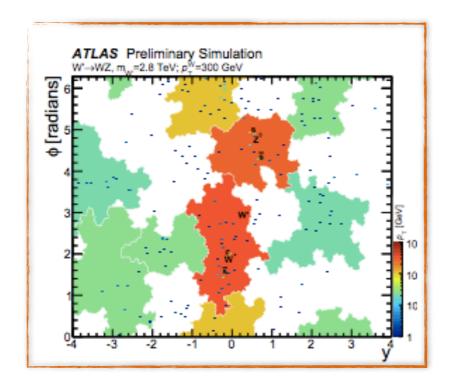


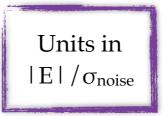
- * Corrugated stainless steel clad Pb absorber sheets,1—2 mm thick
- * Immersed in liquid argon (90K) (similar principle of gas detectors, but liquid)
- Collect ionisation electrons with an electric field across 2.1 mm liquid Argon drift gap

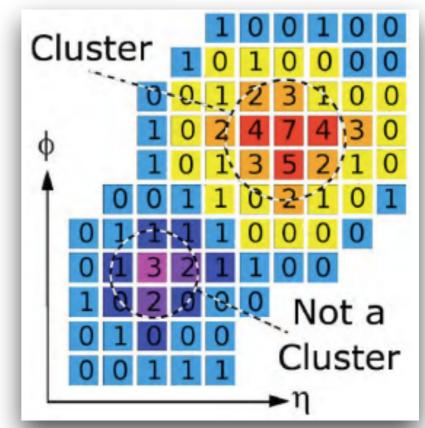
Cluster

$$\rho_{ij} = \min\left(p_{Ti}^{2p}, p_{Tj}^{2p}\right) \left(\Delta R_{ij}\right)^{2} \left(\Delta R_{ij}\right)^{2} + \left(\varphi_{i} - \varphi_{j}\right)^{2}$$

$$\Delta R_{ij}^{2} = \left(\eta_{i} - \eta_{j}\right)^{2} + \left(\varphi_{i} - \varphi_{j}\right)^{2}$$

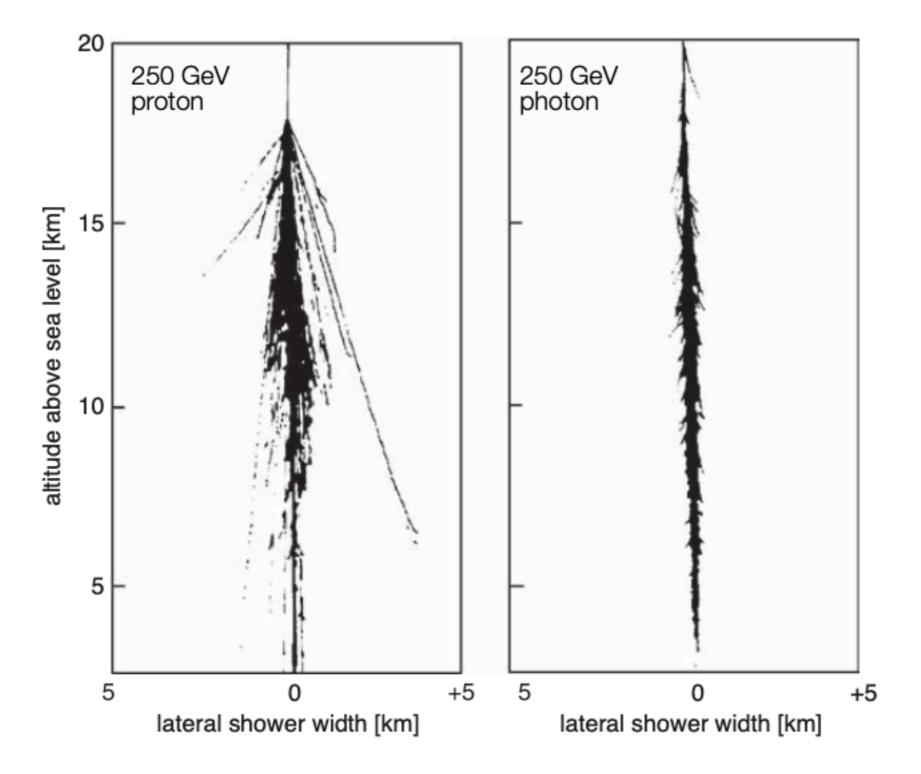






- * Averaged weight on distance and energy deposition in each cell
- * Similar to the center of mass definition

Hadron VS EM

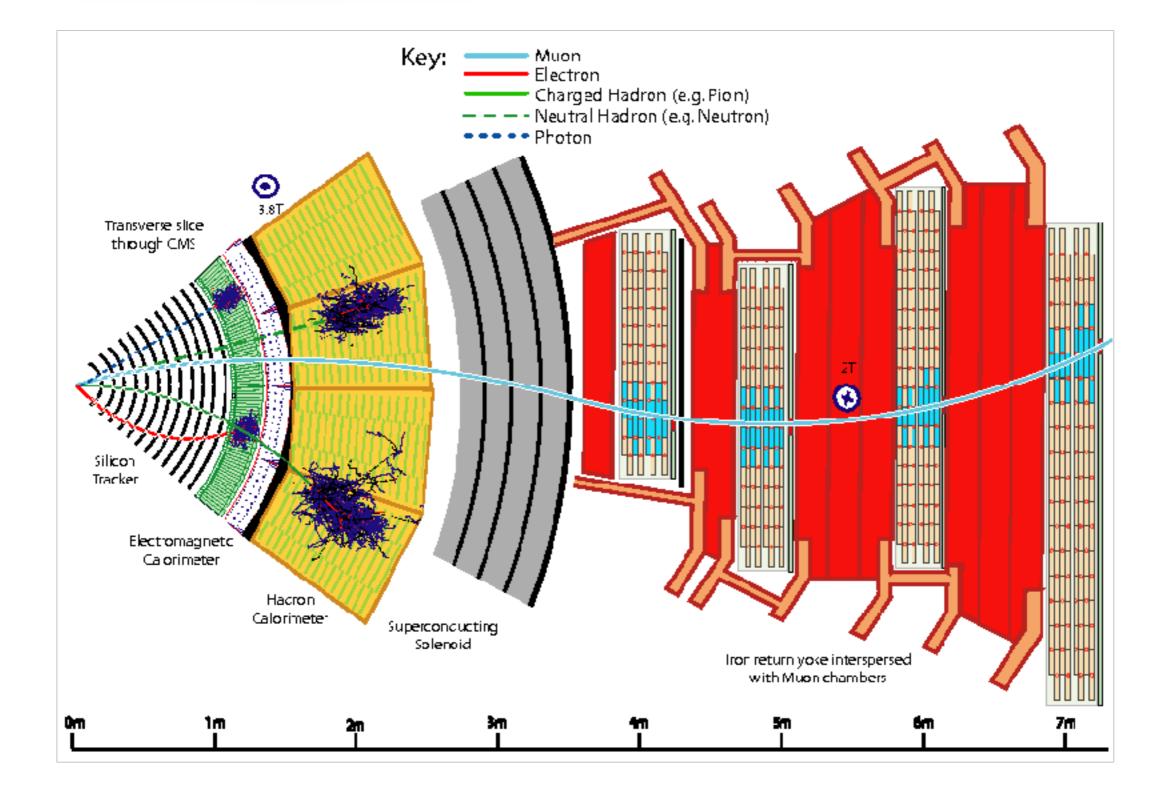


Muon Energy Loss (extra)

Muon Energy Loss

- * Less interacting charged particles.
- * Lose few energy in the detector —> long range
- * Outer detectors to identify them. Match with other detectors to have more precise information.

CMS





ATLAS

